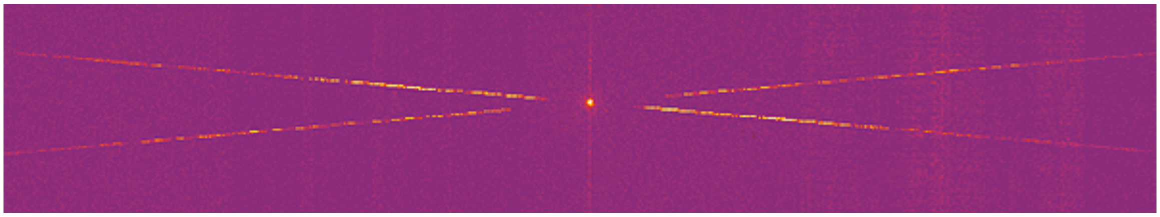
Chandra Data and the Doppler Effect

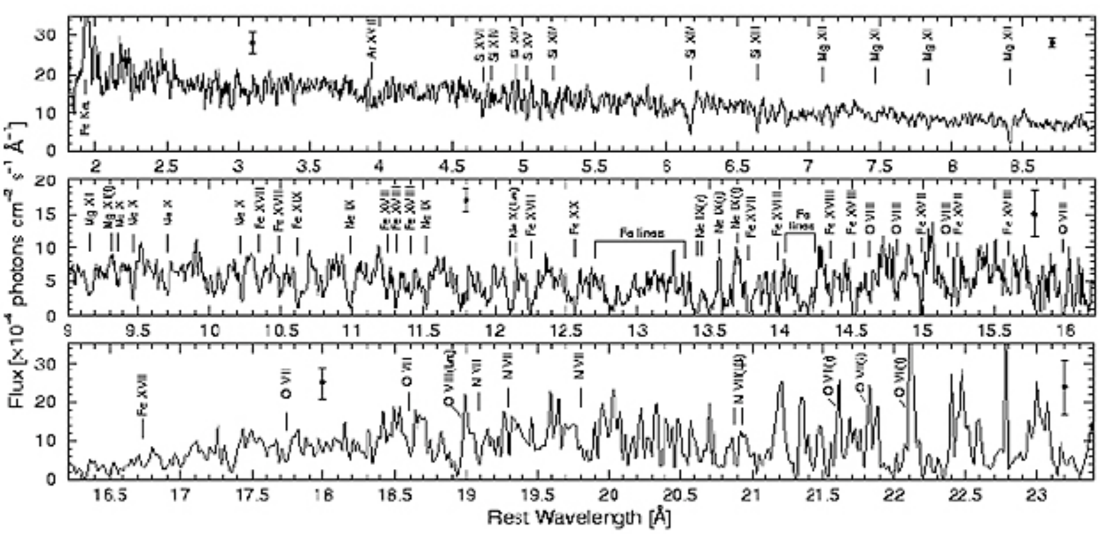


The central bright spot in this image is the Chandra X-ray image of NGC 3783. The long intersecting lines represent dispersed X-ray spectra, or rainbows, produced by the Chandra grating spectrometer. The faint vertical lines are instrumental artifacts. Credit: NASA/PSU

Chandra Clocks Million Mile Per Hour Wind Expanding From Vicinity of Giant Black Hole

NASA's Chandra X-ray Observatory has examined the stormy environs of a giant black hole in the active galaxy NGC 3783 and measured the dramatic effects of intense radiation produced by matter before it plunges into the black hole. This radiation heats surrounding gas and drives a million mile per hour wind away from the crushing grip of the black hole's gravity. ~ Chandra Press Release May 25, 2000

By examining the X-ray spectrum produce by Chandra's High Energy Transmission Grating, scientists were able to measure the velocity of this wind in much the same way a policeman uses a radar gun to determine the speed of a car. A radar gun emits a radio wave of a certain frequency which bounces off the moving car. The detector then receives this echo and determines the velocity from the difference between the frequencies of the emitted signal and its echo.

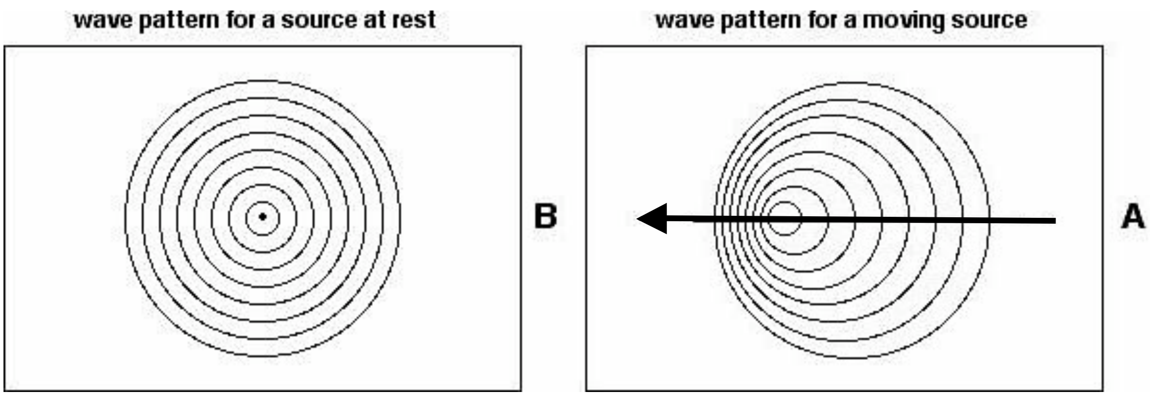


X-ray spectrum showing absorption and emission lines from various elements. Credit: NASA/PSU

In the spectrum of NGC 3783 taken in the vicinity of the black hole, absorption and emission lines produced by specific elements were shifted from their rest wavelengths. The amount of this shift allowed researchers to calculate the velocity of the wind. This shift in the wavelength due to the relative motion of its source is called the **Doppler effect**.

Part A: Visualizing the Doppler Effect

When you drop a rock into a pond, you see a circular wave front expanding in all directions from the impact point. If you were to dip your finger into the water in the same spot repeatedly, at equal time intervals, you would see the waves as a series of equally spaced concentric circles as shown below at the left. If you were to move your finger while doing this, you would see the wave pattern shown below at the right (movement is from A to B).



You may have heard the change in pitch of an ambulance siren or the sound of a race car engine as the vehicle approaches and then passes you. This is due to the compressing of the waves as it approaches and the stretching of waves as it moves away. Consider the equation for wave velocity,

$$\mathbf{v = f \times \lambda}$$

v = wave velocity (speed of sound)
f = frequency (cycles/second or hz)
λ = wavelength

1. If the speed of sound is constant and the wavelength is longer, the frequency is lower. If the wavelength is shorter, the frequency is higher. A higher frequency corresponds to a higher pitch and vice versa.

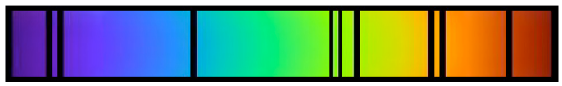
2. How would the pitch of a vehicle's siren change as it approached and then passed you. Why?

3. Light also has wave properties like sound. Red light has a longer wavelength than blue light. How would a galaxy be moving with respect to an observer if its spectrum were red shifted? How would it be moving if its spectrum were blue shifted? Explain your answer.

4. Summarize what you've learned about the Doppler effect by comparing the placement of the lines in the absorption spectra to those in the reference spectrum. Circle the correct choices in bold print. (In case you are viewing a copy of this page in black and white, the red end of the spectrum is on the right.)

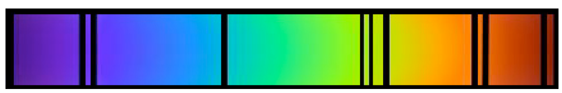
reference absorption spectrum, object is at rest

The absorption spectrum below is **blue/red**-shifted, the frequency of a spectral line is **higher/lower** than its rest frequency, and its wavelength is **longer/shorter** than the rest wavelength.



object is moving towards the observer

The absorption spectrum below is **blue/red**-shifted, the frequency of a spectral line is **higher/lower** than its rest frequency, and its wavelength is **longer/shorter** than the rest wavelength.



object is moving away from the observer

Part B: Quantifying the Doppler Effect

In part A, you saw circular wave fronts expanding from the point of origin for both a source at rest and a moving source. We are now going to draw this quantitatively.

The following variables will be used:

wave period, $\mathbf{T}$
rest wavelength, $\boldsymbol{\lambda_o}$
observed wavelength, $\boldsymbol{\lambda}$
wave velocity, $\mathbf{v}$
source velocity, $\mathbf{v_s}$

Materials:

1/4" graph paper and compass OR templates provided

Note: If you wish to skip steps #1 through #3 where you make 5 drawings to represent wave sources moving at various velocities, use the templates provided at the end of this document.

wave source at rest:

1. On a piece of graph paper, draw a line down the middle of the paper going the long way. Put a point midway along the line where the wave source will be located. Let

$$\boldsymbol{\lambda_o} = 4 \text{ squares}$$
$$\mathbf{v} = (4 \text{ squares})/T$$

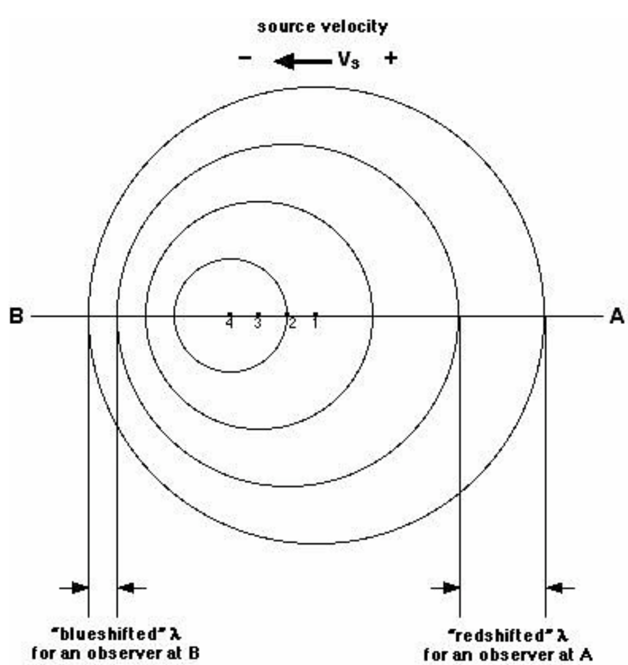
One circular wavefront is emitted from the source each period, T. To represent an elapsed time of 4T, draw four concentric circles around the wave source with radii of 4, 8, 12 and 16 squares. Label this drawing $\mathbf{v_s = 0}$.

moving wave source:

2. Now let's have the source move with a velocity, $\mathbf{v_s = 0.25\ v}$. On a new piece of graph paper, again draw a line down the center. This time the four wave fronts will each have a different point of origin. Since the source velocity is one quarter of the wave velocity,

$$\mathbf{v_s} = \mathbf{0.25v} = .25^*\ (4 \text{ squares})/T = 1 \text{ square}/T$$

Draw four points one square apart on your center line. Number them 1, 2, 3 and 4 from right to left as shown in the figure below. Around point 1, draw a circle with a radius of 16 squares. Repeat for points 2, 3, and 4 with circles of radii 12, 8 and 4 squares respectively. Label this drawing $\mathbf{v_s = 0.25\ v}$.



3. Repeat #2 for drawings with $\mathbf{v_s = 0.50\ v}$ (points 2 squares apart), $\mathbf{v_s = 0.75\ v}$ (points 3 squares apart), and $\mathbf{v_s = 1.00\ v}$ (points 4 squares apart).

4. Let the velocity of an object that is moving away from an observer at point A be positive and towards point B be negative. On each drawing, count the number of squares between wave fronts for an observer at B ("blue shifted" wavelength) and for an observer at A ("red shifted" wavelength). Calculate each observed wavelength, λ, as a multiple of λ_o by dividing the number of squares for the new wavelength by four squares for the rest wavelength. Record these wavelengths in the Data Table.

DATA TABLE

v_s	rest λ
0	1.00 λ_o

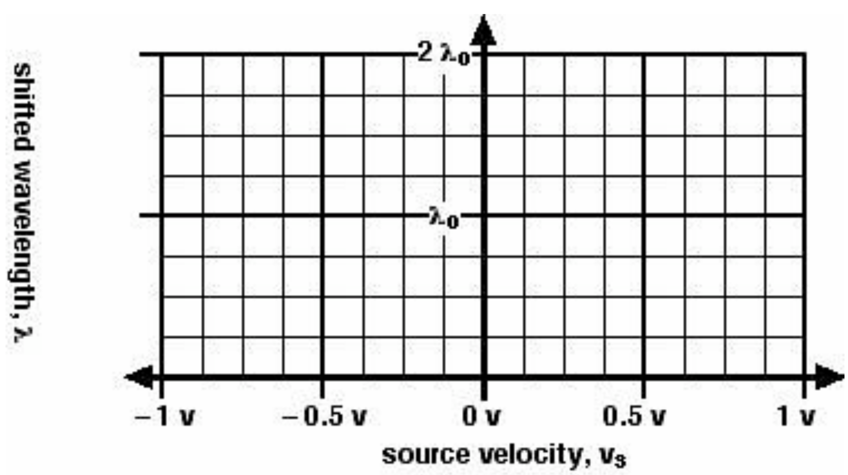
wavelengths for an observer at B

v_s	blueshifted λ
– 0. 25 v	_____ λ_o
– 0.50 v	_____ λ_o
– 0.75 v	_____ λ_o
– 1.00 v	_____ λ_o

wavelengths for an observer at A

v_s	redshifted λ
0.25 v	_____ λ_o
0.50 v	_____ λ_o
0.75 v	_____ λ_o
1.00 v	_____ λ_o

5. Graph your data below, drawing a best fit line through the points.



6. Find the equation of the line using the slope-intercept form, y = mx +b, where

   - **$y = \lambda$**
   - **$x = v_s$**
   - **m = slope** (calculate this from two points on your graph, write as some number times λ_o/v)
   - **b = y-intercept** (read the value of λ at $v_s = 0$ from your graph, write as some number times λ_o.)

Congratulations! You have derived the Doppler formula for wavelength.

7. For light, the velocity of the wave, **v**, becomes **c**, the speed of light (3.0×10^8 m/s). The ratio $\mathbf{\Delta\lambda/\lambda_o}$ ($\mathbf{\Delta\lambda}$ is the change in wavelength, $\mathbf{\lambda - \lambda_o}$) is called the **z parameter** for characterizing red shift. Show how the equation you found in #6 is equal to the equation below.

Doppler shift equation for light

$$z = \Delta\lambda/\lambda_o = v_s/c$$

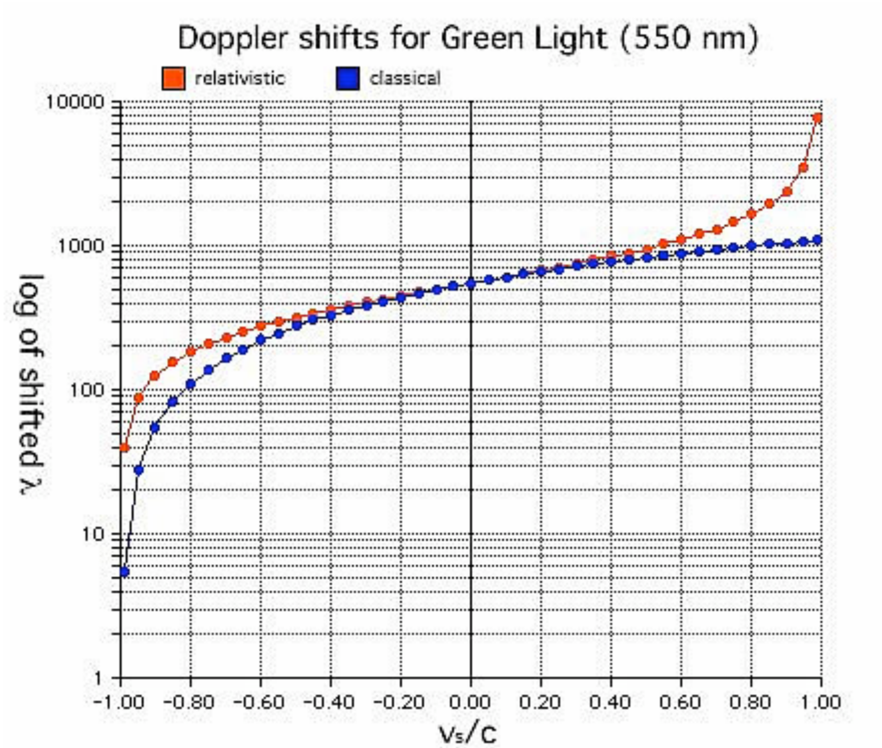
Part C: Comparison of Relativistic and Classical Doppler Effects

According to Einstein's Theory of Special Relativity, a stationary observer would measure the length of an object moving close to the speed of light as shorter than if the object were at rest. When a light source is moving at these speeds, relativistic effects must be taken into account when calculating the Doppler shift since wavelengths become compressed.

relativistic Doppler shift	classical Doppler shift
$z = \Delta\lambda/\lambda_0 = \sqrt{\frac{1 + v_s/c}{1 - v_s/c}} - 1$	$z = \Delta\lambda/\lambda_0 = v/c$
$\lambda = \lambda_0 \sqrt{\frac{1 + v_s/c}{1 - v_s/c}}$	$\lambda = \lambda_0 (1 + v_s/c)$

v_s is negative for an approaching light source and positive for a receding one

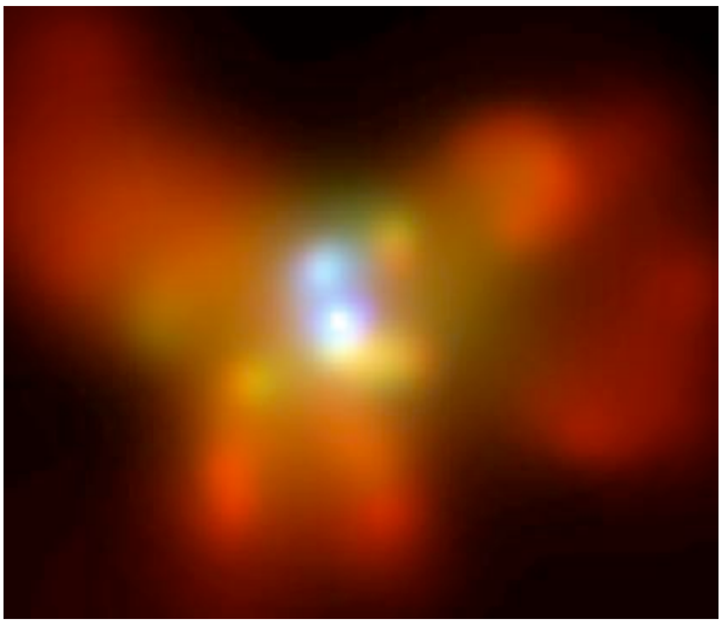
1. The following graph compares the Doppler shifts predicted by classical mechanics and relativity. For a moving source of light, what are the ranges of its velocity relative to the observer (as a fraction of the speed of light) for which relativistic effects can be ignored?



Part D: Hubble's Law

Astronomers compute the distance to remote galaxies (ones that are more than about 20 million light years away) with Hubble's law. According to Hubble's law, the universe is expanding in such a way that distant galaxies are receding from one another with a speed, **v**, which is proportional to their distance, **d**. The recession causes the radiation from a galaxy to shift to longer wavelengths—the red shift. From a measurement of the red shift and the constant of proportionality, called Hubble's constant, $\mathbf{H_o}$, astronomers can determine the distance to a galaxy.

$$\mathbf{v = H_o \text{ X } d} \qquad \textbf{Hubble's Law}$$



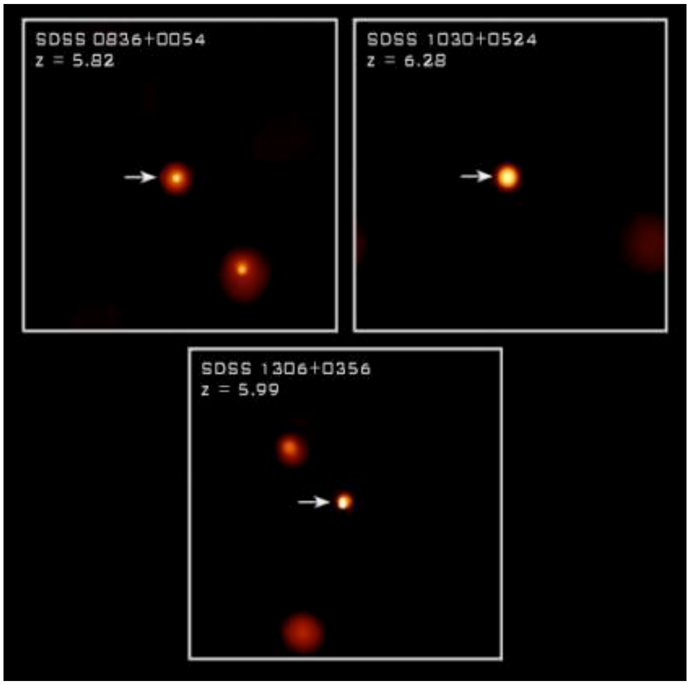
The Chandra image of NGC 6240, a butterfly-shaped galaxy that is the product of the collision of two smaller galaxies, revealed that the central region of the galaxy (inset) contains not one, but two active giant black holes. Credit: NASA/CXC/MPE/ S.Komossa et al.

1. Using Hubble's Law and a value for $\mathbf{H_o}$ of 65 km/s/Mpc, calculate the recession velocity (in km/s) of NGC 6240 (shown above) which is about 400 million light years away. (1 Mpc = $3.26\text{X}10^6$ light years)

2. Referring to your results in Part C, is this a galaxy for which you would need to take relativistic effects into account? What is the red shift parameter, z, for this galaxy?

3. The hydrogen-a spectral line has a rest wavelength, λ_o, of 6562.8 Å (1Å = 1 X 10^{-10} m) Use the Doppler shift equation (see the table in Part C) to calculate the observed wavelength of this line for the spectrum of NGC 6240.

Quasars are small but very bright objects with large red shifts indicating that they are very far away. They radiate as much energy per second as a thousand or more galaxies, from a region that has a diameter about one millionth that of the host galaxy. Quasars are intense sources of X-rays as well as visible light. They are the most powerful type of X-ray source yet discovered. Some quasars are so bright that they can be seen at a distance of 13 billion light years!



These three quasars, recently discovered at optical wavelengths by the Sloan Digital Sky Survey, are 13 billion light years from Earth, making them the most distant known quasars. The X-rays Chandra detected were emitted when the universe was only a billion years old, about 7 percent of the present age of the universe.
Credit: NASA/CXC/PSU/N.Brandt et al.

4. Consider the data below for three of the most distant known quasars detected by Chandra. For these extremely high redshifts, relativistic effects must be taken into account. Find the recession velocity of each quasar as a fraction of the speed of light. The relativistic Doppler equation solved for v/c is

$$v/c = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

quasar	**redshift**	**v/c**
SDSS 0836+0054	$z = 5.82$	_______________
SDSS 1030+0525	$z = 6.28$	_______________
SDSS 1306+0356	$z = 5.99$	_______________

One of the central problems of modern astronomy is to accurately determine Hubble's constant, H_o, which is a measure of the rate of expansion of the universe. At present it is known to an accuracy of about 20 percent.

Read the this Chandra press release: Flickering Quasar Helps Chandra Measure the Expansion Rate of the Universe.

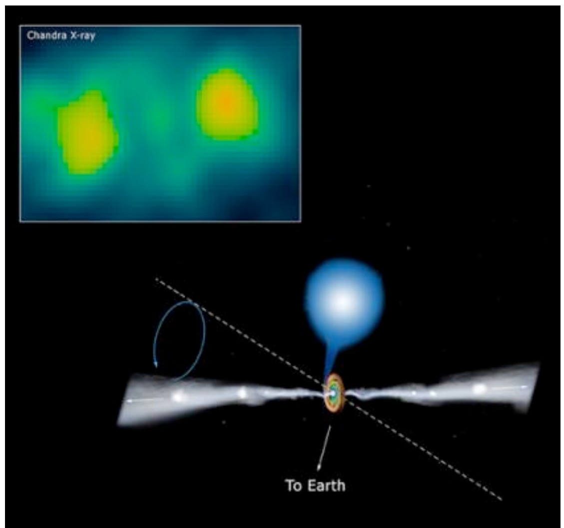
5. How can a quasar provide an absolute distance to a galaxy acting as a gravitational lens between it and earth?

6. How can Chandra improve this process?

7. How could this data be used to estimate the expansion rate of the universe?

Part E: Examining the Jets of SS 433

SS 433 is a binary system consisting of a blue supergiant and a black hole. It lies approximately 16,000 light-years away in Aquila within the 40,000-year-old supernova, W50. The black hole and its companion are about two-thirds closer to each other than Mercury is to the Sun. Material transfers from the main sequence star onto a disk of hot material surrounding the black hole. Material is ejected from this disk in narrow high speed jets of ionized gas that slowly wobble or precess around a circle (represented by blue circular arrow), from the sketched location of the jet at one extreme to the dotted white line at another.



SS 433: A black hole & massive star binary star system about 16,000 light years from Earth (Credit: X-ray: NASA/CXC/U.Amsterdam/ S.Migliari et al., Illustration: CXC/M.Weiss)

These high speed jets may be caused by the same mechanisms as in more distant and more massive black holes such as quasars. SS 433 provides a local laboratory in which to study the conditions in such jets. In addition, details in how the jets change with time and distance from the black hole can contribute to new techniques with which to find the mass of the black hole.

Observations indicate that one lobe is moving along a line tilted toward Earth whereas the other one is moving along a line tilted away from Earth. By measuring Doppler shifts in x-ray spectral lines in hot spots in both the blue-shifted and red-shifted lobes, the velocity of material in the jets, $\mathbf{v_j}$, can be calculated.

Since the jets are not moving along the line of sight to Earth, red and blue shifts give only the component or part of this velocity along the line of sight. These jets also move at a significant fraction of the speed of light so the Doppler formula must take into account relativistic effects as well as the angle between the direction of the jets and the line of sight (α). This Doppler shift formula is as follows (if $\alpha = 0$, it can be shown that this formula reduces to the relativistic formula given at the beginning of Part C.):

$$\mathbf{z = \gamma(1 \pm \beta\cos\alpha)} \text{ where } \boldsymbol{\gamma = (1 - \beta^2)^{-1/2}} \text{ and } \boldsymbol{\beta = v_j/c}$$

The blue-shift, $\mathbf{z_b}$, is found by using the – sign and the red shift, $\mathbf{z_r}$, by using the + sign. We can obtain an estimate of $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ without knowing the angle between the jet direction and line of sight by adding the red and blue shifts to cancel the $\boldsymbol{\beta\cos\alpha}$ term.

$$\boldsymbol{\gamma = [(z_b + z_r)/2] + 1}$$

8. Use a spreadsheet and the data in **Table 1** to calculate the average blue-shift, $\mathbf{z_b}$, and red shift, $\mathbf{z_r}$. Remember, $\mathbf{z = \Delta\lambda/\lambda_o = (\lambda_{obs} - \lambda_{rest}) / \lambda_{rest}}$.

Table 1. SS 433 X-ray Emission Lines

Blue Jet			Red Jet		
λ_{rest} (Å)	λ_{obs} (Å)	Identification	λ_{rest} (Å)	λ_{obs} (Å)	Identification
1.1780	1.641	Fe XXVI	1.592	1.836	Ni XXVII
1.855	1.710	FE XXV	1.780	2.057	Fe XXVI
3.020	2.795	Ca XX	1.855	2.147	FeXXV
3.186	2.935	Ca XIX	6.182	7.133	Si XIV
3.733	3.432	Ar XVIII	6.675	7.747	Si XIII
3.952	3.650	Ar XVII	7.986	9.214	Fe XXIV
3.991	3.682	S XVI			
4.299	3.959	S XV			
4.729	4.362	S XVI			
5.055	4.675	S XV			
5.217	4.825	Si XIV			
6.182	5.703	Si XIV			
6.675	6.169	Si XIII			
7.986	7.370	Fe XIV			
8.310	7.663	Fe XXIII/XXIV			
8.421	7.756	Mg XII			
9.102	8.370	Ne X			
9.181	8.495	Mg Xi			
10.368	9.552	Ne IX			
10.634	9.795	Fe XXIV			
11.008	10.159	Fe XXIV			
11. 176	10.303	Fe XXIV			
11.736	10.827	Fe XXIII			
12.134	11.194	Ne X & FeXXIII			

9. Using your averge $\mathbf{z_b}$ and $\mathbf{z_r}$, calculate $\boldsymbol{\gamma}$

$$\boldsymbol{\gamma = [(z_b + z_r)/2] + 1}$$

and then $\boldsymbol{\beta}$.

$$\boldsymbol{\gamma = (1 - \beta^2)^{-1/2}} \text{ so } \boldsymbol{\beta = (1 - 1/\gamma^2)^{1/2}}$$

At what fraction of the speed of light do Doppler shifts indicate that the jets are moving?

The following is from a December 10, 2002 Chandra press release regarding SS 433:

Lobes of unexpectedly hot gas speeding away from a black hole in our galaxy have been discovered by NASA's Chandra X-ray Observatory. The high temperature and the distance of the lobes from the black hole indicate that violent collisions are occurring between clumps of gas expelled from the vicinity of the black hole.

"Just like a super highway, it's a dangerous world out there," said Simone Migliari on the University of Amsterdam, lead author on a paper from a September 6, 2002 issue of Science magazine. "Blobs of gas are getting rear-ended at speeds in excess of a hundred million miles per hours!"

10. Do your results agree with the information given in the last paragraph? Show a calculation to support your answer. Read more on SS 433 in the January 05, 2004 MIT press release.

11. Read one of the articles below and write a summary of what else scientists have discovered using the Doppler effect.

   - The Terrible Twos: What Might Happen If Our Sun Had A Twin
   - Scientists Observe Light Fighting To Escape Black Hole's Pull
   - Chandra Finds Oxygen and Neon Ring in Ashes of Exploded Star
   - Chandra Discovers the X-ray Signature of a Powerful Wind from a Galactic Microquasar