Unit I Reading: Significant Figures

Laboratory investigations usually involve the taking of and interpretation of measurements. All physical measurements obtained by means of instruments (meter sticks, thermometers, electrical meters, clocks, etc.)are to some extent uncertain. If, for example, the mass of an object is determined by means of a Dial-O-Gram balance, the measured mass will be uncertain by at least $\pm$ 0.01 gram. If the object were now weighed on progressively more accurate scales, the uncertainty in the mass of the object would get progressively less, but regardless of the precision of the measuring device, any instrumental measurement is to some extent uncertain. The degree of uncertainty in physical measurements can be indicated by means of significant figures. Consider, for example, a measurement of the length of the object as indicated below, with three differently calibrated meter sticks.

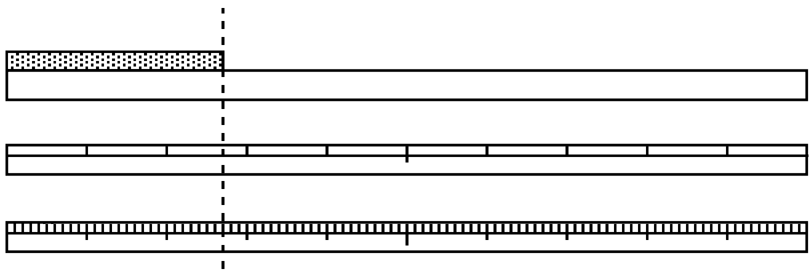

Figure 1

Observe that when measuring the length of the object with the uncalibrated meter stick (top) the actual length of the object in Figure 1 can only be estimated, and then only to the nearest tenth of a meter, or as 0.3 meter (one significant figure).

Measuring the length of the object, however, with a meter stick calibrated in tenths of a meter (center stick in Figure 1) it is obvious that the length of the object is greater than 0.2 m but less than 0.3 m. Once again, it would seem to be reasonable to estimate the length of the object to the nearest tenth of the smallest calibration or to the nearest hundredth of a meter; thus 0.27 m. It might actually be as short as 0.26 m or as long as 0.28 m, so 0.27 m (to the nearest hundredth of a meter) seems to be the most reasonable estimate of the object's length. This measurement has two significant figures indicating less uncertainty in the second measurement than in the first.

Measuring the length of the object with a meter stick calibrated in hundredths of a meter (lower stick in figure 1), the length of the object could be estimated to tenths of the smallest calibrations (centimeters) or the measured length could be estimated to the nearest millimeter; nearer to 0.270 m than to 0.269 m or 0.271 m. Note that this measurement has three significant figures indicating less uncertainty in this measurement than in either of the other two preceding measurements. Thus, the number of significant figures in a measurement indicates the precision of the measurement and not the absolute length of the object.

Once the logic of significant figures is accepted, some simple rules are useful for their implementation.

Rule 1- which digits are significant: The digits in a measurement that are considered significant are all of those digits that represent marked calibrations on the measuring device plus one additional digit to represent the estimated digit (tenths of the smallest calibration).

The zero digit is used somewhat uniquely in measurements. A zero might be used either as an indication of uncertainty or simply as a place holder. For example, the distance from the earth to the sun is commonly given as 1,500,000,000 km. The zeroes in this measurement are not intended to indicate that the distance is accurate to the nearest km, rather these zeroes are being used as place holders only and are thus not considered significant.

Rules for zeros:

1. All non-zero digits in a measurement **are** considered to be significant.
2. Zeroes **are** significant if bounded by non-zero digits; e.g., the measurement 4003 m has four significant figures.
3. If a decimal point is expressed, all zeroes following non-zero digits **are** significant; e.g., the measurement 30.00 kg has four significant figures.
4. If a decimal point is not explicitly expressed, zeroes following the last non-zero digit **are not** significant, they are **place holders only**; e.g., the measurement 160 N has two significant figures.
5. Zeroes preceding the first non-zero digit **are not** significant , they are **place holders only**; e.g., the measurement 0.00610m has three significant figures.

As an example, take the process of finding the average of the following series of measurements:

$t_o = 20.78$ s
$t_1 = 20.32$ s
$t_2 = 20.44$ s
$t_3 = 21.02$ s
$t_4 = 20.81$ s
$t_5 = 20.63$ s
$t_6 = 21.12$ s

$$t_{av} = (t_o + t_1 + t_2 + t_3 + t_4 + t_5 + t_6) \div 7 = 20.73 \text{ s}$$

The rule developed earlier in this discussion suggested that we should retain, as significant figures, all digits those values we were certain of plus one estimated digit. With this rule, we would retain the digit in the tens column because all of the data values in this column are the same (we are certain of these values). We would also retain the digit in the units column because, even though there are some differences in this column, the rule says we may retain one digit that is estimated (value of the digit in this column is uncertain).

The rule then suggests that we should retain only 2 digits (tens and units) for t_{av}, and after rounding, the best value would be $t_{av} = 21$ s.

Rules for addition and subtraction with significant figures:

1. Change the units of all measurements, if necessary, so that all measurements are expressed in the same units (kilograms, meters, degrees Celsius, etc.).
2. The sum or difference of measurements may have no more decimal places than the least number of places in any measurement.

   For example:

   11.44 m
   5.00 m
   0.11 m
   <u>13.2 m</u>
   29.750 m

   But since the last measurement (13.2 m) is expressed to only one decimal place, the sum may be expressed to only one decimal place. Thus 29.750 m is rounded to 29.8 m.

   Consider the quotient: 294,921 cm^2 ÷ 38 cm. What should the answer be?
   8,000 cm, or 7,800 cm, or 7,760 cm, or 7,761 cm?

   The question is what uncertainty do we wish to express in a product or quotient? To answer this question we might wish to examine the above example. Recall that the last digit in each measurement is an estimated digit so the product might be as large as 7,970.86 cm (maximum value), or as small as 7,562.05 cm (minimum value).
   Observe that while the digits in the thousands column are both the same, the values of the digits in the hundreds column vary. Therefore, the quotient would be 7,800 cm, to two significant figures. Note that the number of figures in the quotient is the same as the least number of significant digits in either the divisor or the dividend. If we were to test many examples, we would find this relationship to hold true in most cases, leading to the following rule.

Rules for multiplication and division with significant figures:

Students typically make one of two mistakes: either they keep *too few* figures by rounding off too much and lose information, or they keep *too many* figures by writing down whatever the calcualtor displays. Use of significant figure rules helps us express values with a reasonable amount degree of precision.

When multiplying or dividing, the number of significant figures retained may not exceed the **least** number of digits in either the of the factors.

Example: 0.304 cm x 73.84168 cm. The calculator displays 22.447871. A more reasonable answer is 22.4 cm^2 . This product has only three significant figures because one of the factors (0.304 cm) has only three significant figures, therefore the product can have only three.

Another example: 0.1700 g ÷ 8.50 L. The calculator display of 0.02 g/L, while numerically correct, leaves the impression that the answer is not known with much certainty. Expressing the density as 0.0200 g/L leaves the reader with the sense that very careful measurements were made.