A topless roller coaster

Chapter 6 – extra credit

This MUST be done alone and is due the day of the test. It is worth up to 20 bonus points.

From "Quantoons: Metaphysical Illustrations" by Tomas Bunk, A. Eisenkraft and L. Kirkpatrick.

"Designing an amusement park ride is quite a challenge. The ride must be entertaining *and* safe. Modern roller coasters have added a new dimension to roller coaster riding by adding such things as loops and corkscrews. These require our readers, as future designers, to apply knowledge of circular motion and centripetal acceleration in addition to the conservation of mechanical energy in analyzing the rides. Of course, in order to do this in a simple manner way we can make a number of assumptions such as (1) there are no frictional forces (including air resistance), (2) the kinetic energy of the wheels can be neglected, (3) the train of the roller coaster cars stays on the track without the safety rail, and (4) the train is a point mass. The last assumption allows us to neglect such things as the rotational kinetic energy, the angular momentum, and the orientation of the train.

Up to now, all of the roller coasters of the world use a continuous track. But that does not restrict our imagination. Let's imagine we remove the top portion of the track in a vertical loop, creating the so-called topless roller coaster. This allows us to combine the physics of circular motion in a gravitational field with that of **projectile motion**.

Assume that the vertical loop is a circle with a radius R and that the portion that is missing has an angle of 2α as shown in figure 1."

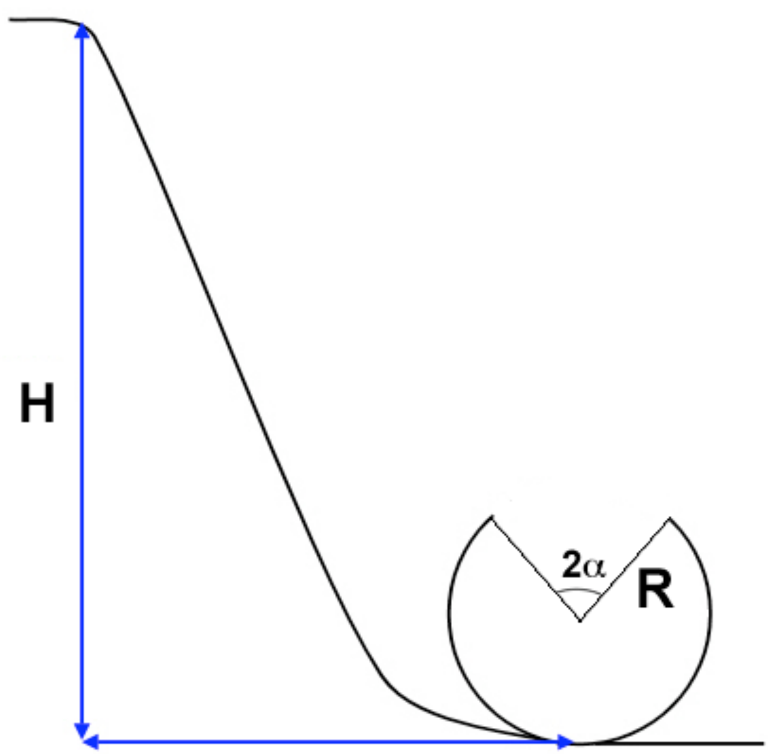


Figure 1.

Show all work neatly!!!!

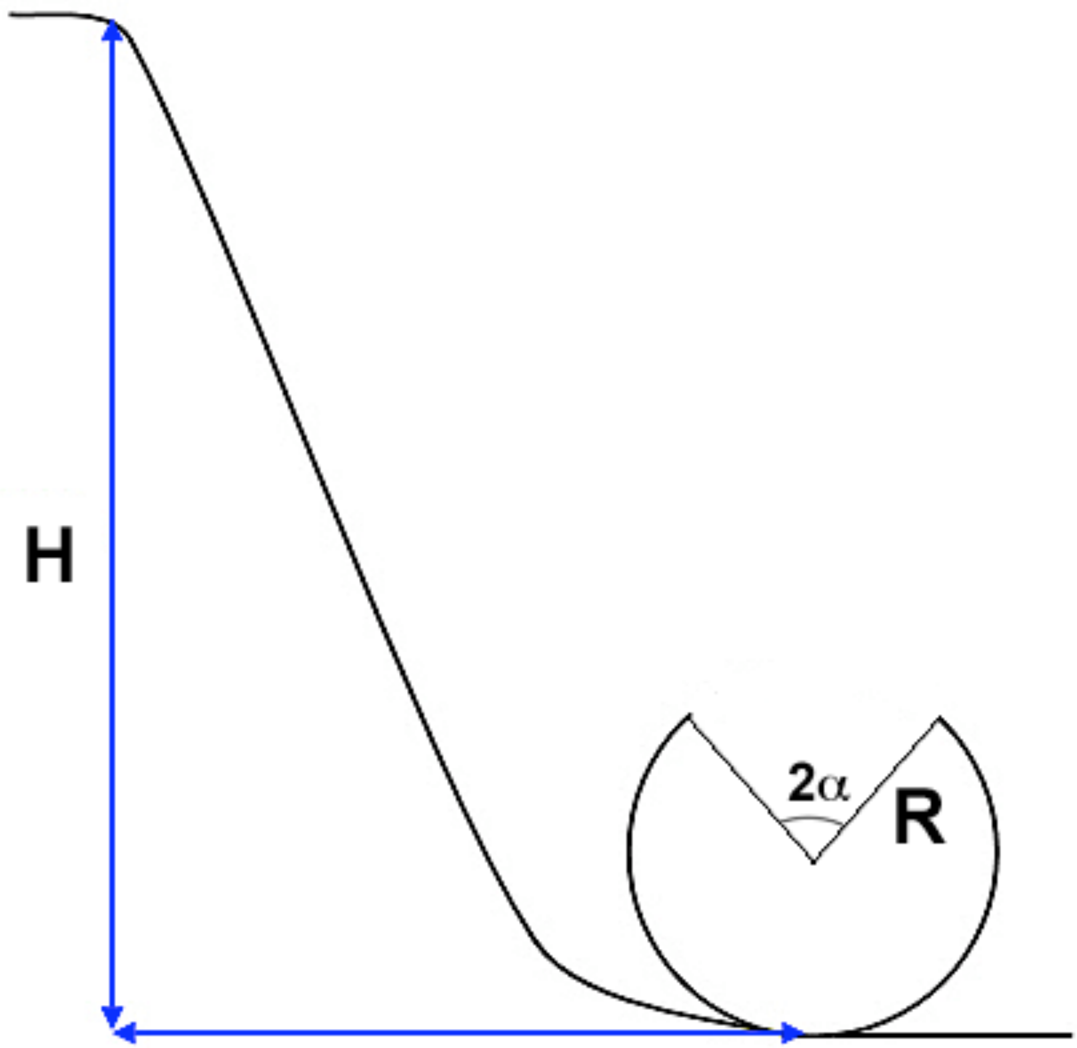
1. First pretend that the loop is closed. Using conservation of energy and centripetal acceleration, find the minimum ratio H/R that would allow the coaster train to make it around the top point on the loop without falling out. (Note: For the minimum velocity at the top of the loop, the normal force on the train by the track would be zero).

2. Now, for the topless coaster, find an equation for H/R in terms of angle, α. Use conservation of energy at the point where the track stops and combine that equation with the one for range of a projectile (the range must be large enough to cross the opening). This problem will require that you use lots of geometry to figure out heights and angles, etc. I've included a larger version of figure 1. on the last page on which you can draw. Remember the trig identity, $\sin 2\alpha = 2\sin\alpha\cos\alpha$.

3. For the topless coaster, graph H/R vs α in degrees.

4. For what range of angle is the coaster possible if the height is restricted to $H < 3R$?

5. Using your equation from #2, what is H/R if $\alpha = 0^{\circ}$? How does this compare to your answer for #1? For what other angle do you get this same ratio?


H
2α
R