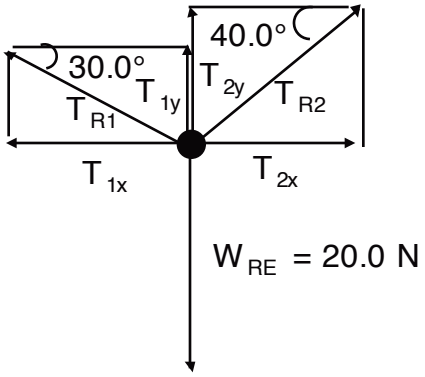
tension - unequal angles

Find the tensions in strings 1 and 2 supporting a 20.0 N rock hanging from the ceiling. Strings 1 and 2 make angles of 30.0° and 40.0° with the ceiling, respectively.



Var	Given value	Units	Description
θ_1	30.0	°	angle of string 1 with ceiling
θ_2	40.0	°	angle of string 2 with ceiling
W_{RE}	20.0	N	weight on rock by earth
$F_{NET,X}$	0	N	sum of forces in x–direction
$F_{NET,Y}$	0	N	sum of forces in y–direction
$T_{R\,1}$		N	tension on rock by string 1
$T_{1\,x}$		N	x–component of T_{R1}
$T_{1\,y}$		N	y–component of T_{R1}
$T_{R\,2}$		N	tension on rock by string 2
$T_{2\,x}$		N	x–component of T_{R2}
$T_{2\,y}$		N	y–component of T_{R2}

Equation for the sum of forces in the x–direction:

tension - unequal angles (continued)

$$F_{NET,X} = 0$$

$$-T_{1x} + T_{2x} = 0$$

$$-T_{R1}\cos\theta_1 + T_{R2}\cos\theta_2 = 0$$

Equation for the sum of forces in the y–direction:

$$F_{NET,Y} = 0$$

$$T_{1y} + T_{2y} + (-W_{RE}) = 0$$

$$T_{R1}\sin\theta_1 + T_{R2}\sin\theta_2 + (-W_{RE}) = 0$$

We have 2 equations and two unknowns. Solve the first equation for T_{R1}.

$$-T_{R1}\cos\theta_1 + T_{R2}\cos\theta_2 = 0$$

$$T_{R1}\cos\theta_1 = T_{R2}\cos\theta_2$$

$$T_{R1} = \frac{T_{R2}\cos\theta_2}{\cos\theta_1}$$

Substitute for T_{R1} into the second equation and solve for T_{R2}.

tension - unequal angles (continued)

$$T_{R1}\sin\theta_1 + T_{R2}\sin\theta_2 + (-W_{RE}) = 0$$

$$\frac{T_{R2}\cos\theta_2}{\cos\theta_1}\sin\theta_1 + T_{R2}\sin\theta_2 + (-W_{RE}) = 0$$

$$\frac{T_{R2}\cos\theta_2}{\cos\theta_1}\sin\theta_1 + T_{R2}\sin\theta_2 = W_{RE}$$

$$\left(\sin\theta_2 + \frac{\cos\theta_2}{\cos\theta_1}\sin\theta_1\right) T_{R2} = W_{RE}$$

$$T_{R2} = \frac{W_{RE}}{\sin\theta_2 + \frac{\cos\theta_2}{\cos\theta_1}\sin\theta_1}$$

$$= \frac{20.0\,\text{N}}{\sin(40.0°) + \frac{\cos(40.0°)}{\cos(30.0°)}\sin(30.0°)}$$

$$= 18.4\,\text{N} \quad ✓$$

$$T_{R1} = \frac{T_{R2}\cos\theta_2}{\cos\theta_1}$$

$$= \frac{(18.4\,\text{N})\cos(40.0°)}{\cos(30.0°)}$$

$$= 16.3\,\text{N} \quad ✓$$